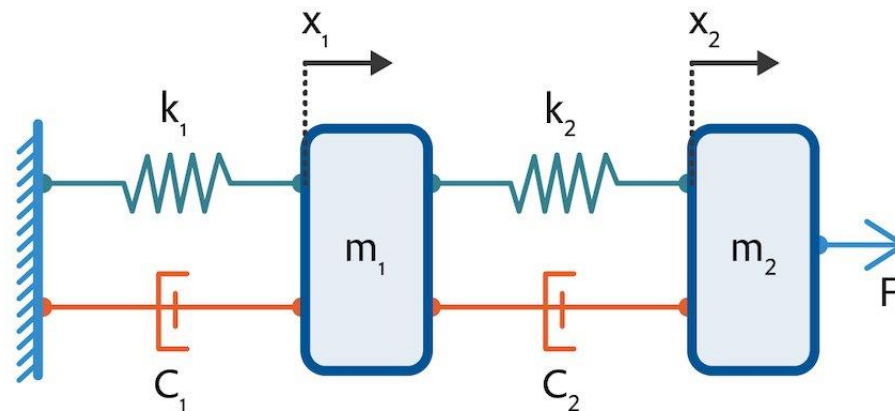
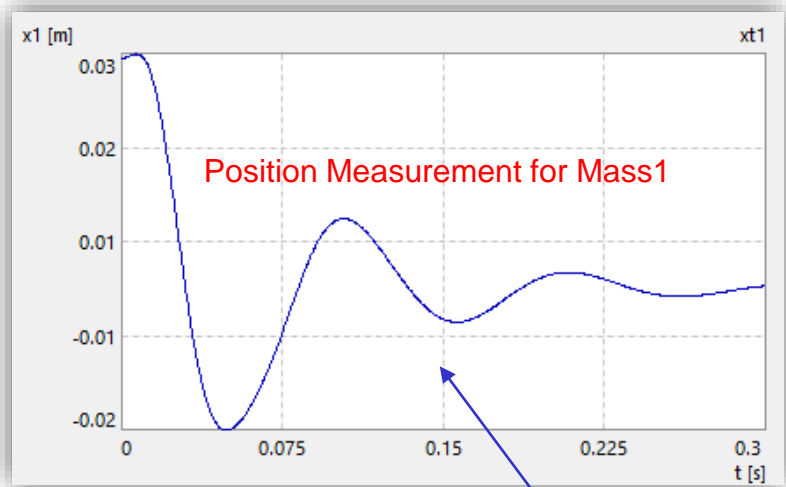


Novel Technology Case Study

Data-Driven Modeling and Simulation of a double Spring Mass Damper System



Imperfect Data and Physics of the System

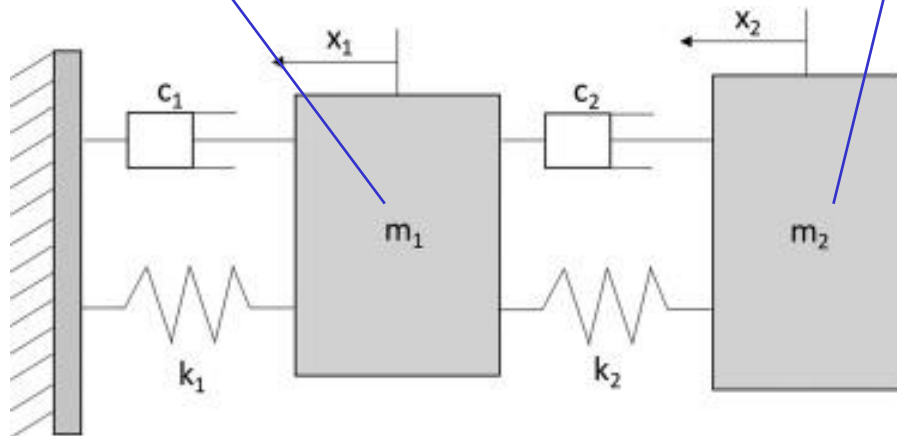


Partial Differential Equation for Mass 2

$$m_2 \ddot{x}_2 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) = 0$$

Initial Conditions for Mass 2

$$x_2 = 0.055 \quad \dot{x}_2 = 0$$



System-Modeling using Mix: Data + Physics

Hilbert Space	
Include X-Axis	<input checked="" type="checkbox"/>
Include 1D-Variables	<input type="checkbox"/>
X-Nonlinearity	None
Uniform Space	<input type="checkbox"/>
t	
Covariance Function	Exponential
Number of Features	20
Gaussian Noise [%]	0.01
Approximation Error [%]	0
Regularization Weight	1
Include DOE	<input checked="" type="checkbox"/>
Weight	1
Partial Differential Equatio	<input type="checkbox"/>
Boundary Conditions	<input type="checkbox"/>
Constraints	<input type="checkbox"/>
Parameters	<input type="checkbox"/>

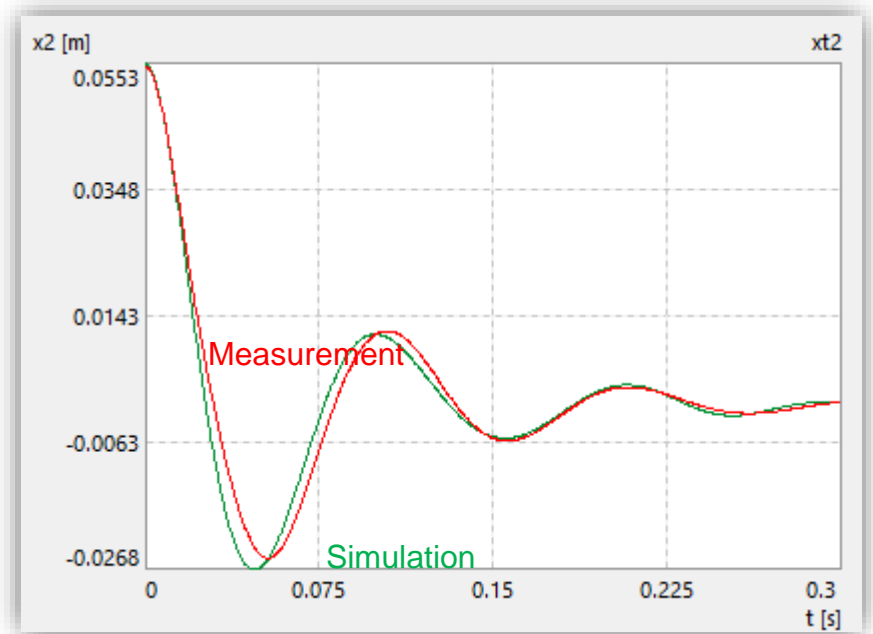
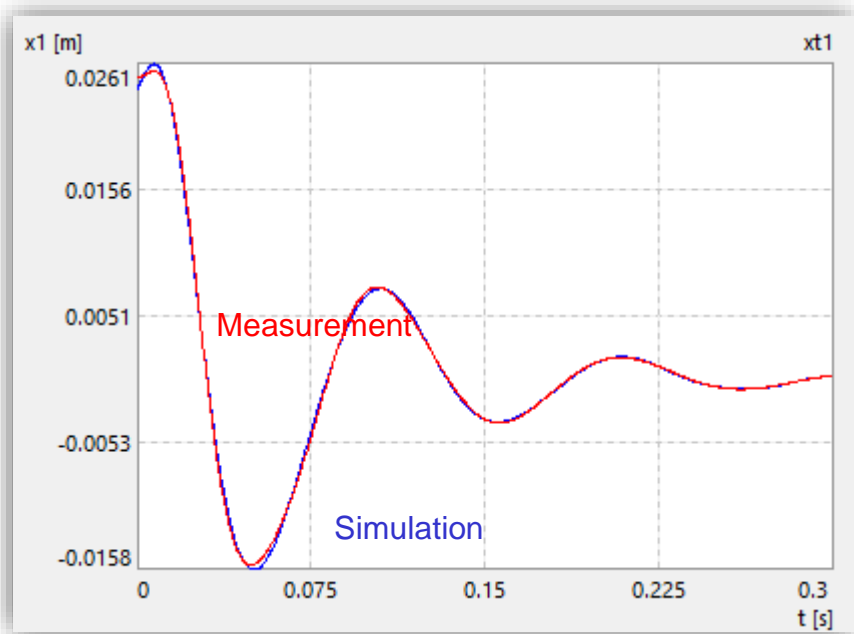
Only Data for Mass 1

Hilbert Space	
Include X-Axis	<input checked="" type="checkbox"/>
Include 1D-Variables	<input type="checkbox"/>
X-Nonlinearity	None
Uniform Space	<input type="checkbox"/>
t	
Covariance Function	Exponential
Number of Features	15
Gaussian Noise [%]	0.01
Approximation Error [%]	0
Regularization Weight	1
Include DOE	<input type="checkbox"/>
Partial Differential Equation	
PDE	$m2/k2*derivate(xt2,t,t)+c2/k2*derivate(xt2,t)-c2/k2*derivate(xt1,t)+xt2-xt1=0$
Linear	<input checked="" type="checkbox"/>
Sampling Level	5
Weight	1
Boundary Conditions	
Number of Boundaries	2
Boundary 1	
Initial Value	$xt2=0.055$
Number of fixed Values	1
Fixed Parameter	t
Sampling Level	5
Weight [0..1]	1
Boundary 2	
Initial Value	$derivate(xt2,t)=0$
Number of fixed Values	1
Fixed Parameter	t
Sampling Level	5
Weight [0..1]	1
Constraints	<input type="checkbox"/>
Parameters	<input type="checkbox"/>

Differential Equation for Mass 2

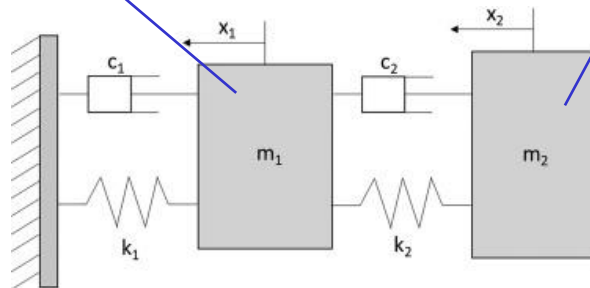
Initial Conditions for Mass 2

Real-Time Simulation and Measurement



Position for Mass 1

Position for Mass 2



Summary

- For the modeling system, there are sometime in the reality only **imperfect data** from measurement and **imperfect physics** in form of partial differential equation, initial condition or constraints. The goal is modeling and simulation of the total system behaviours
- **Physics-informed machine learning** (PIML) is the best technology to solve this problem. Based on the Hilbert space, the meta-model can be modeled from both data and physics representing the system response.
- For the double spring mass damper system, there are only measurement data for the position of mass 1 and partial differential equation and initial conditions for mass 2. From these imperfect data and physics, the system response can be modeled and simulated in real-time. The simulation and measurement of the system response coincides totally.